# A Critical Lunar Eclipse Observation by Āryabhaṭa-I 

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Salient features of the total lunar eclipse of 23 March 517 CE is examined in contrast to the discussion on eclipses given by Āryabhata in his work Āryabhaț̄ya. Āryabhata's description of the range of colors that the moon's disc exhibits during a total lunar eclipse is shown to conform with the modern descriptions and the Danjon scale of lunar eclipse brightness. Comparison is made of the computational results according to Āryabhatīya and modern astronomy. Moment of full moon and totality deduced by the Aryabhata elements tally precisely with modern astronomical value. This close agreement despite the deficiency of inequalities in $\bar{A} r y a b h a t i \bar{y} y a$ and the precise agreement of mean longitudes of sun and moon suggests that the eclipse under reference may have been a critical input for Āryabhaṭa in giving final shape to his treatise.

Also discussed is the observation of the meridian transit of the totally eclipsed moon and the method of Āryabhaṭa tradition for finding the meridian ecliptic point as well as the transit of moon. This observation is shown to have facilitated an accurate verification of the true longitude of both moon and sun on 23 March 517 CE. Mean solar ingress of Kali 3618 elapsed as per Āryabhat̄̄ya, 21 March 517 CE, 06:00 ZT, Kalidina of 1321506 and the true equinox are shown to have an intervening span of 2.13 days which precisely correspond to the equation of centre of $\bar{A} r y a b h a t a$. Result in turn suggest that the equinox of 517 CE and the meridian transit of the eclipsed moon may have been reference to Aryabhaṭa in the formulation of $\bar{A} r y a b h a t \bar{\imath} y a$.

## I. Introduction

Recent studies ${ }^{1,2}$ have brought to light the precise date and place of Āryabhaṭa as well as the important solar eclipse observations that formed the basis of the elements of Āryabhatīya. ${ }^{3}$ Exact concurrence of the Moon's Node and Apogee of Āryabhaṭīya with modern astronomical elements for CE 519 substantiate the Haridatta tradition of Kerala that takes the Kali year Girituňga (3623) as the epoch of the treatise. Āryabhaṭa's treatment of eclipses in Golapāda mainly consisted of the verses 33-34 giving the parallax necessary in solar eclipse computation and verses $38-46$ present the computational details of the lunar eclipse. Verse 47 refer to the visibility of partial solar eclipses. Explicit treatment of the fixing of apparent conjunction with Lambana-nāḍikās we miss in Āryabhaṭīya and this has led to the interpretation of verse 38 of Goḷapāda as referring to parvānta as the middle of the solar and lunar eclipse ${ }^{4}$. Precise computation of the parallax had been a difficult exercise in his times and as such the test of a Siddhānta, mainly, the luni-solar elements could be done best with eclipses centered over the meridian where the parallax is zero. This may be the reason for considering the middle of the eclipse as close to the parvānta in solar eclipse.

## Lunar Eclipse Observation by Āryabhaṭa - Verse 46 of Golapāda

In the case of solar eclipse, the obscured disc appears black during all phases of the eclipse while in a lunar eclipse, the obscured disc undergo changes in color from smoky, black, tawny and . Āryabhaṭa described a lunar eclipse in verse 46 of Goḷapāda as -

> प्रग्रहणान्ते धूम्रः खण्डग्रहणे शशी भवति कृष्णः। सर्वग्रासे कपिलः सकृष्णताम्रो तमो मध्ये ।।
"At the beginning and end of lunar eclipse, the obscured disc appears smoky and during partial obscuration it is black. In totality the obscured part is yellowish brown and at maximum obscuration the disc appears bluish with black tinge"
Obviously, the verse reflects the observation of a total lunar eclipse by Āryabhata and the description of the phases in different colours reminds us of the Danjon scale of lunar eclipse brightness ${ }^{f}$. Earth's atmosphere refracts some of the Sun's rays into the shadow depending upon the filtering by atmospheric materials. Therefore the total phase of a lunar eclipse has a vibrant range of colours from dark brown and red to bright orange and yellow. The exact appearance depends on how much dust and clouds are present in Earth's atmosphere. A modern picture as below ${ }^{5}$ we find reflected in the above verse of Āryabhaṭa.

Fig. 1: Range of Colors of Moon's disc in a Total Lunar Eclipse


When we look for the specific total lunar eclipse that was visible at his place in the vicinity of the solar eclipses of CE 519, we meet with the following cases shown in table-1:

[^0]Table-1: Lunar Eclipses Visible at Camravattam (10N51, 75E45)

| Date | Zone Time <br> 10N51, 75E45 | Partial / <br> Total | Remarks | Julian Days <br> (ZT) |
| :---: | :---: | :---: | :---: | :---: |
| 516 September 26 | $00: 40$ | Partial | -- | 1909795.52778 |
| 517 March 23 | $23: 37$ | Total | On the Meridian <br> \& Clear Sky | 1909974.48403 |
| 517 September 15 | $03: 40$ | Total | Visible partially | 1910149.65278 |
| 520 January 20 | $23: 04$ | Partial | -- | 1911007.46111 |
| 521 January 8 | $04: 15$ | Total | Visible partially | 1911360.67708 |
| 521 July 5 | 23:00 | Total | During heavy <br> Monsoon | 1911539.45833 |

We can find that the total lunar eclipse of 23 March 517 CE would have been quite special for an astronomer observing the sky over the meridian of Ujjayinī as the eclipse totality occurred on the meridian. Even though the lunar eclipse of 5 July 521 is total and totality occurred close to the meridian, it happened during heavy Monsoon and is unlikely to be observed due to rain/clouds. Total lunar eclipse of 15 September 517 even though visible took place after the meridian transit and therefore it may have been visible at the place of A Aryabhata.
Further, the total lunar eclipse of 23 March 517 CE took place on Citrā paurnamī night when the Moon was in conjunction with Citrā and at the totality of the eclipse the Citrā star must have been visible on the meridian. Right ascension ( $\alpha$ ) and declination of the Moon and Citrā for midnight of 23/24 March 517 CE point towards a longitudinal conjunction.

$$
\begin{array}{lll}
\text { Citrā or } \alpha \text {-Virginis: } \alpha=12 \mathrm{~h}-09 \mathrm{~m} & \delta=-03^{0} 05^{\prime}, \text { Upper transit: } 23: 54 \\
\text { Eclipsed Moon } & \alpha=12 \mathrm{~h}-17 \mathrm{~m} & \delta=-02^{0} 13^{\prime}, \text { Upper transit: 00:06 }
\end{array}
$$

It is therefore evident that during the totality of the eclipse both Moon and the star Citrā had the meridian transit and therefore Āryabhata had the opportunity to have a check of the longitude of Moon as well as zero parallax by comparing it with the longitude of Citrā.

## Importance of Meridian Transit of Moon in Āryabhaṭa Tradition

Importance of the meridian observations of the Moon in the Āryabhata tradition may be understood from the three verses by Bhāskara-I dedicated to the description of the same and the necessary computations: ${ }^{6}$

## आसन्नौ स्वधिया $f$ भ्यूह्य मध्यलग्ननिशाकरौ। कालेन्दुमध्यलग्रानामविशेषं समाचरेत्।।

"Approximate time of conjunction of the Moon with the meridian ecliptic point is first determined and then successively the Moon and meridian ecliptic point are computed till the agreement is reached i.e the values become equal"

## नाङ्योन्तरालजाः साध्या लङ्काराश्युदयैस्तयोः। ऊने विश्लेषनाडीभ्यः शुद्धिः क्षेपोfधिके रमृतः ।।

"With rising times of signs at Laňkā, find the time that is needed for Moon to equal the meridian ecliptic point. Correct the Moon accordingly and repeat the process by assuming the meridian ecliptic point and find the time when the rising point and the moon increased by three signs are equal"

> मध्यलग्नसमश्चन्द्रो जायतेfनेन कर्मणा । तत्क्षिप्त्यपक्रमाक्षैस्तु मध्यछाया प्रसाध्यते ।।
"This is how the Moon's longitude at meridian transit is obtained. From moon's celestial latitude and declination, the zenith distance cane be found out for any place using the latitude"
Further, last but second verse with which the astronomical contents of the Āryabhaṭīya have been concluded, Gola 48 highlights the importance of lunar eclipses (earth-sun conjunctions) in fixing the computation of sun. ${ }^{7}$

$$
\begin{align*}
& \text { क्षितिरवियोगात् दिनकृत् रवीन्दुयोगात् प्रसाधितश्चेन्दुः | } \\
& \text { शशिताराग्रहयोगात् तथैव ताराग्रहाः सर्वे ।| } \tag{48}
\end{align*}
$$

"Sun is computed from the earth-sun conjunctions, Moon is computed from its conjunctions with the sun and the star-like planets are computed by observing their conjunctions with the moon"

Remarkable features as above of the total lunar eclipse of 23 March 517 CE and especially the meridian transit during totality and maximum makes it an interesting observation that Āryabhaṭa may have utilized in giving final shape to his treatise.

## II. Lunar Eclipse of 23 March 517 CE

The total lunar eclipse of 23 March 517 CE happened just after the opening of the new year and when the sun had just crossed the equator to the north. Kali year 3618 had its beginning on 21 March 517 CE with the mean solar ingress into Aries. Day and night were equal on the date of the eclipse and thus sunset and meridian transit of Moon presented reliable time markers for the astronomer.

Elements of Āryabhaṭiya and modern algorithms ${ }^{8}$ renders the following computational results (table-2) for the total lunar eclipse of 23 March 517 CE. Delta T ( $\delta$ T = TT-UT) for 517 CE turns out to be 4503 seconds. Julian Dates have been given for the longitude of 75 E 45 , same as that of the native place of Āryabhaṭa and Ujjayinī and is given as JD (ZT), ZT meaning Zonal Time at 75E45. Kalidina is computed from the zero epoch defined as 18 February - 3101CE, 06:00 ZT corresponding to Kalidina $=0$ and $\mathrm{JD}(\mathrm{ZT})=588465.75$ days.

Table-2: Contrast of Āryabhaṭīya and Modern Results

|  | Kalidays | $\begin{gathered} \hline \text { Full Moon } \\ \mathrm{JD}^{1} \\ \hline \end{gathered}$ | $\begin{gathered} \text { Mean } \\ \lambda \mathrm{s} \end{gathered}$ | True $\lambda s$ | $\begin{gathered} \text { Mean } \\ \lambda \mathrm{m} \end{gathered}$ | True $\lambda \mathrm{m}$ | $\begin{gathered} \lambda \\ \text { node } \end{gathered}$ | $\begin{gathered} \lambda \\ \text { apogee } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Āryabhaṭīya Mean Full Moon | 1321508.301 | 1909974.051 | $02^{0} 22^{\prime}$ | $04^{0} 27^{\prime}$ | $182^{0} 22^{\prime}$ | $178^{0} 46$ ' | $183^{0} 44^{\prime}$ | $48^{0} 17{ }^{\prime}$ |
| Modern $\lambda$ |  |  | $02^{0} 24^{\prime}$ | $04^{0} 18{ }^{\prime}$ | 182 ${ }^{0} 23$ ' | $178^{0} 16^{\prime}$ | $183^{0} 43^{\prime}$ | $48^{0} 12^{\prime}$ |
| Mod. Mean Full Moon | 1321508.302 | 1909974.0523 | $02^{0} 24^{\prime}$ | $04^{0} 18^{\prime}$ | $182^{0} 24^{\prime}$ | $178^{0} 18^{\prime}$ | $183^{0} 43$ ' | $48^{0} 12$, |
| Āryabhatīya True Full Moon | 1321508.737 | 1909974.4866 | $02^{0} 47^{\prime}$ | $04^{0} 52$, | $188^{0} 06{ }^{\prime}$ | $184^{0} 52^{\prime}$ | $183^{0} 44^{\prime}$ | $48^{0} 17$ |
| Modern True Full Moon | 1321508.742 | 1909974.4922 | $02^{0} 50$ | $04^{0} 44^{\prime}$ | $188^{0} 12^{\prime}$ | $184^{0} 44^{\prime}$ | $183^{0} 42^{\prime}$ | $48^{0} 15$ |

Data as above is illustrative of the extreme accuracy that Āryabhaṭa achieved. The following points are note worthy.
(a) Time of mean full moon as per Āryabhaṭīya and modern algorithm differed only by 2 minutes.
(b) Mean longitudes of Sun and Moon matched precisely with the corresponding modern values.
(c) Time of true full moon as per Āryabhaṭīya was only 8 minutes less of modern value.
(d) True $\lambda$ of sun as per Āryabhatīya differed from modern value by (+)9 minutes of arc while true $\lambda$ moon of Āryabhaṭīya differed from modern value by $(+) 30$ minutes of arc at the Mean Full Moon epoch. But at the true Full Moon, the computed mean and true $\lambda \mathrm{s}$ were in close agreement.
(e) Node and apogee of Moon precisely matched with the modern values. This in turn reinforces the conclusions derived from examining the solar eclipses of 15 Feb and 11 Aug 519 CE in the earlier study given at reference (2).
(f) Inequality for Sun as per Āryabhaṭīya is $128^{\prime}$ and the modern value is $115^{\prime}$. It is therefore evident that the error in the true value is due to the high value of the equation of centre adopted in Āryabhaṭ̄ya.

[^1]Perfect agreement of the Siddhāntic computational results as above with the modern astronomical values render support to the fact that the treatise had its origin around the epoch of the lunar eclipse of 23 March 517CE.

## Extreme Accuracy of the Sun, Moon Longitudes at Full Moon

Extreme accuracy of the luni-solar mean and true longitudes at the full moon is an indirect pointer towards the methodology of Āryabhaṭa. It may be noted that the equation of centre for sun according to Āryabhata is $128.8^{\prime}$, surplus by 13 ' compared to the modern value of -115.5 minutes of arc and in the case of moon the contrast is far more striking with the Aryabhata value of $300.1^{\prime}$ and modern equation of $377.2^{\prime}$. Despite such apparent discrepancy, Āryabhaṭa's method is giving extremely accurate values for the full moon computed.

The early Indian astronomers failed to detect the inequalities excepting the equation of centre as the mean longitudes of sun and moon and equation of centre were derived by the analysis of the times of eclipses which occur only at syzygies. Annual equation arising from the distance of earth and moon from sun amounting to $+11 \sin$ (sun-anomaly) of moon got added to sun as the full moon times were based on the elongation (moon - sun) i.e. $-115^{\prime} \sin \mathrm{m}-11^{\prime} \sin \mathrm{m}=$ $128.8^{\prime}$ max as we see in Āryabhaṭiya. In the case of full moon, the line of apsides explains 30' error in true $\lambda$ moon. Āryabhațīya equation of centre does not include evection and in the present case where the line of apsides is nearly $45^{\circ}$ from the luminaries, evection is almost halved to $38^{\prime}$ and therefore the A Aryabhaṭīya equation of centre is less by the same amount and hence the surplus by $30^{\prime}$ in true $\lambda$ moon.

The fact that Aryabhata failed to notice the evection and variation despite his mysterious accuracy conveys us that the astronomer used only the eclipse times as cross check of his computations. As corollary, we can argue that if the eclipse of 23 March 517CE had been a reference for Āryabhaṭa for giving shape to Āryabhaṭiya, then the timings of the eclipse derived from the method must match precisely with the modern astronomical times.

## Salient Features of Lunar Eclipse Discussion in Āryabhaṭīya

1. True Full Moon: 23 March 517CE, 23:41 ZT, JD(ZT)=1909974.4866
2. Primary elements include the mean diameters and distances of Sun, Moon and Earth which are furnished below: Also given are the true distances and diameters computed as per the precepts of Aryabhata ${ }^{9}$ in table- 3 below:
3. Modern astronomical times of totality and duration have been taken from the Sky Map pro Planetarium software.

Table-3: Mean and True Distances and Diameters of Sun and Moon

| Element | Sun | Earth | Moon |
| :--- | :---: | :---: | :---: |
| Mean Diameter | 4410 | 1050 | 315 |
| Mean Distance | 459585 | 0 | 34377 |
| True Diameter | 4369 | 1050 | 334 |
| True Distance | 460538 | 0 | 36757 |

4. Latitude $\beta$ of Moon $=0^{0} 06^{\prime}$
5. Moon's daily motion $=837.34^{\prime}$ and Sun's daily motion $=58.5886$.
6. True diameter of Sun in minutes $=32.62$ and true diameter of Moon in minutes $=31.21$
7. Length of the Earth's shadow

Āryabhaṭīya Gola 39 instructs the computation of the length of the Earth's shadow:

> भूरविविवरं विभजेत् भूगुणितं तु रविभूविशेषेण । भूच्छायादीर्घत्वं लब्धं भूगोलविष्कम्भात् ।।
"True distance of the sun multiplied by the diameter of earth and divided by the difference of diameters of the sun and earth gives the length of the shadow of earth from the earth's centre"

Length of the earth's shadow $=$ [Sun's distance*Earth Dia/(Sun Dia-Earth Dia)] $=145689$ Yojanas $=4.2$ times the distance of moon from earth.

According to modern astronomy, at the mean distance of the earth from Sun, the length of the earth's shadow will be more than 3 times the distance of moon from the earth. Earth's shadow will be of maximum size i.e. greatest length and breadth when the sun is at aphelion. At the time of full moon, moon shall cross the shadow at the point decided by its nearness to earth and ancient astronomers could understand the geometry well by the 'lamp and shadow' ${ }^{10}$ method.

## 8. Diameter of the earth's shadow at Moon's distance is given by verse 40 of Gola: <br> छायाग्रचन्द्रविवरं भूविष्कम्भेण तत् समभ्यस्तम् । भूच्छायया विभक्तं विद्यात् तमसः स्वविष्कम्भम् II

"Difference of the length of earth's shadow and the distance of moon multiplied by the earth's diameter and divided by the earth's shadow gives the diameter of Tamas i.e. diameter of the earth's shadow at moon's distance"
Diameter of earth's shadow at Moon's distance $=$ Diameter of Tamas $=$ [Length of earth's shadow-Moon's distance]*Earth's Diameter/Length of earth's shadow] $=785.1$ Yojanas

## 9. Semi-dia of earth's shadow at Moon's distance $=\mathbf{3 9 2} .50$ Yojanas $=36.72$ '

This angular breadth of the half shadow may be computed using modern algorithm as $\quad \mathrm{P}+$ $\pi-\psi$ where P is the horizontal parallax of moon, $\pi$ the parallax of sun ( $8^{\prime \prime}$ ) and $\psi$ the semidiameter of sun. In the present case for the time of first contact $\mathrm{P}=53^{\prime}$ and $\pi-\psi=16^{\prime}$ and hence the semi-diameter of the shadow $=53^{\prime}-16^{\prime}=37^{\prime}$, very nearly the same.
10. Half duration of the lunar eclipse is given by verse 41 of Gola.

तच्छशिसम्पर्कार्धकृतेः शशि विक्षेपवर्गितं शोध्यम् । स्थित्यर्धमरय मूलं गेयं चन्द्रार्कदिनभोगात् ।।
"Square the sum of semi-diameters of Tamas and Moon and subtract the square of moon's latitude and root of this gives the semi-duration of the eclipse in minutes. Using the tithi-gathi the corresponding time in ghatis can be found".
$\sigma=($ Shadow + Moon $)=52.32^{\prime}$.
Half duration $=\left[60^{*}\left(\sigma^{\wedge} 2-\beta^{\wedge} 2\right)^{\wedge} 0.5\right] /(\delta \lambda \mathrm{s}-\delta \lambda \mathrm{m})=96$ minutes $=1 \mathrm{~h} 36 \mathrm{~m}$

## 11. Half duration of totality - Verse 42 of Gola

> चन्द्रव्यासार्धोनस्य वर्गितं यत्तमोमयार्धस्य । विक्षेपकृतिविहीनं त स्मान्मूलं विमर्दार्धम् ।।
"Square the difference of semi-diameters of Tamas and Moon and subtract the square of moon's latitude and root of this gives the semi-duration of the totality of the eclipse in minutes.".

Here too using the tithi-gathi the corresponding time in ghatis can be found. $s=$ semi-diameter of Tamas - semi-diameter of Moon $=21.11, \beta=6.05^{\prime}$.

Half duration of totality $=\left[60^{*}\left(s^{\wedge} 2-\beta^{\wedge} 2\right) / 778.75\right]=1.56$ ghațis $=37$ minutes

## 12. Un-obscured part of the Moon's disc - Verse 43 of Gola तमसो विष्कम्भार्धं शशिविष्कम्भार्धवर्जितमपोह्य । विक्षेपाद्यच्छेषं न गृह्यते तच्छशाङ्कर्य ।।

"From the semi-diameter of Tamas subtract the moon's semi-diameter and the result deducted from the latitude of Moon gives the un-obscured portion of the moon's disc"

Unobscured part of the Moon $=(\beta-s)=15^{\prime}=48 \%$
This result substantiates the assumption of parvānta or the moment of true full moon as more or less the middle of the eclipse in verse 38 of Golapāda.

## Comparison with Modern Results

Based on the time of full moon and half duration of the eclipse computed as above, the eclipse times as per Āryabhaṭīya and the modern algorithms are give below in table-4:

Table-4: Eclipse times - Modern and Āryabhaṭa Results

| Eclipse | Modern | Āryabhaṭa |
| :---: | :---: | :---: |
| Duration of the total phase | 1 h 39 m | $37 \times 2=1 \mathrm{~h} 14 \mathrm{~m}$ |
| Duration of Umbral phase | 3 h 34 m. | $96 \times 2=3 \mathrm{~h} 12 \mathrm{~m}$ |
| Maximum of eclipse | $23: 37 \mathrm{ZT}$ | $23: 41 \mathrm{ZT}$ |
| Start of Totality | $22: 48$ | $23: 04$ |
| End of Totality | $00: 26$ | $0: 18$ |

Results according to Āryabhatīya and modern astronomy are in close agreement. We saw above that the time of full moon as per Āryabhaṭīya and modern astronomy differed only by 2 minutes of time.
It must be noted here that the Āryabhatịiya does not mention the corrections such as Udayāntara ${ }^{11}$, Bhujāntara (equation of time) and Deśāntara and in this examination we have contrasted the modern results with those of Āryabhatịya devoid of the three corrections referred above. Udayāntara and Bhujantara are irrelevant as we have computed the time of full moon by iteration instead of working with longitudes for local sunrise at 75E45, 10N51. Deśāntara had no impact as Āryabhaṭīya takes the local meridian 75E45 which is equal to that of Ujjayinī as reference.

Eclipsed Moon transited the meridian in totality with copper color and as compared to other Indian Latitudes, at the Kerala latitudes moon had been closer to zenith and facilitated accurate measurement of zenith distance. At Kanyākumāri both the moonrise and moonset could be watched against the sea horizon and at 10 N 51 the setting moon could be observed against the sea horizon for precise measurements and refining of the elements.

## Āryabhaṭa Missed Udayāntara and Bhujāntara (Equation of Time)

The fact that Āryabhaṭa did not give correction for Udayāntara and Bhujāntara (equation of time ${ }^{12}$ despite his precision matching modern algorithms is a pointer towards his location close to Laňka ( $0 \mathrm{~N} 0,75 \mathrm{E} 45$ ) on equator whose sunrise had been his reference for fixing time. Observations of Kuppaṇṇa Sāstry are notable in this context: ${ }^{10}$
"In the Saura of Pancasiddhāntikā the Bhujāntara correction appears for the first time, followed by all later astronomers. The Āryabhatīyam does not mention this, but the followers have argued that this is intended in gītik $\bar{a} 2$ by the expression, arkodayācca lan̆kāyām as done
by Govindaswamy under Mahābhāskarīyam IV.7. Its appearance in the Khaṇ̣̣akhādyaka shows this probable. At any rate Bhāskara I gives it in his work IV.7, 24, 29-30"

Discussion by Sāstry brings out the fact that Āryabhaṭa did not give corrections for the equation of time and difference in sunrise between Laňkā and higher latitudes as his specific observations which have gone into the making of the treatise was not affected by such factors. But astronomers of the Āryabhaṭa tradition like Bhāskara-I and Khaṇ̣̣akhādyaka of Brahmagupta had been trying to make out the same from Āryabhatīya in view of the higher latitudes where the obliquity and latitude had been causing a significant difference between the sunrise and the mean sunrise of Laňkā.

Depending on sundial for his time measurement, Āryabhata had to be located close to the equator to miss the equation of time. As for example on 12 February 517 CE the sunrise and therefore the Kalidina of 1321469 correspond to $06: 23$ at 10 N 51 while it is $06: 39$ at 25 N 51 (Kusumapura). Difference with the mean sunrise of Laňkā at 25 N51 is 39 minutes and observations based on 'seasonal hours' (1/12 of day and night) could not have matched with the results computed using Kalidina. Accuracy we see in full moon time on 23 March 517 CE suggests that the original parameters of Āryabhaṭīya are immune to errors possible due to Udayāntara and this is possible only if the treatise is framed out of observations and time reckoning unaffected by Udayāntara. Total lunar eclipse of 23 March 517 CE presented to him the ideal conditions required at 10N51, 75E45 i.e. time reckoning and observations unaffected by Udayāntara and Deśāntara.

## Meridian Transit and sensitivity of time reckoning

As discussed under section I, the meridian transit of the totally eclipsed moon had critical significance for an ancient astronomer engaged in refining his siddhānta. Mahābhāskarīya VI. 39-41 instructs the trial and error method of fixing the Moon and the upper transit which may be illustrated as shown in table- 5 below:

Table-5: Meridian Transit of Eclipsed Moon at 10N51, 75E45

| Trial | Time | Lagna | Midheaven | Moon | Moon $+90^{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $23: 00$ | 254.47 | 167.03 | 184.21 | 274.21 |
| 2 | $23: 30$ | 261.27 | 174.30 | 184.5 | 274.50 |
| 3 | $0: 00$ | 268.09 | 181.61 | 184.79 | 274.79 |
| 4 | $0: 08$ | 270 | 183.65 | 184.87 | 274.87 |
| $\mathbf{5}$ | $\mathbf{0 : 1 4}$ | $\mathbf{2 7 1 . 2}$ | $\mathbf{1 8 4 . 9 3}$ | $\mathbf{1 8 4 . 9 2}$ | 274.92 |
| 6 | $0: 30$ | 274.98 | 188.98 | 185.08 | 275.08 |
| 7 | $1: 00$ | 281.98 | 196.47 | 185.38 | 275.38 |

Verse 39 instructs to have an approximate time like 23:00 or 23:30 and compute Lagna and Moon. Comparing Moon $+90^{\circ}$ with Lagna and mid-heaven we can understand that the moon's meridian transit is later by nearly 17 degrees and a second trial can be made after 1 hour at 00:00 hours and by repeating the trial the exact moment of moon's transit can be found by computation and matched with the observation. In the case of the lunar eclipse of 23 March 517 CE discussed above, moon thus determined as equal to meridian ecliptic point also gave the value of sun as the totally eclipsed moon on meridian obviously indicated sun to be moon $+180^{\circ}$. Moon being of negligible (7') latitude precise matching of the computed ecliptic longitude with actual placement in the sky was possible.

## Fixing the Mean Longitude of Sun

Epoch as above of 23 March 517 CE with its proximity to the vernal equinox facilitated a convenient estimation of the equinoctial shadow and also of the mean longitude of sun subsequently. Spring equinox on 19 March 517 CE, 03:39 ZT differed from the Kalidina of 1321504 days by less than 0.1 day or nearly 6 ghațis or 6 minutes of arc while the Kali year 3618 expired at 1321505.9 i.e. precisely two days after the moment of spring equinox. We may note here that -

- JD (ZT) Spring equinox on 19 March 517 CE 03:39 ZT or 22:36 UT (23:52TT) = 1909969.6524 days
- Kali zero epoch of Āryabhaṭiya $=18$ Feb -3101 CE, 06:00 ZT and JD $(Z T)=588465.75$
- Kalidina up to the spring equinox on 19 March $517 \mathrm{CE}=1909969.6524$ days -588465.75 $=1321503.902$ days.
- (Kalidina of true equinox +2 days) $/$ Kali years $3618=1321505.902 / 3618=365.25868=$ Solar year of Āryabhaṭiya.

We can easily understand the above coincidence by a look at the equation of centre corresponding to the spring equinox. Apogee $=78^{0}$ and the equation of centre will be $128^{\prime} * \sin$ $78^{0}=125.2^{\prime}=02^{0} 05^{\prime}$ and thus we get the following results:

- Mean solar ingress of Kali 3818 elapsed: 21 March 517 CE, 06:00 ZT $=1321506$ days.
- Modern true longitude of Sun for 21 March $517 \mathrm{CE}, 06: 00 \mathrm{ZT}=02^{\circ} 03^{\prime}$
- Equation of centre as per Āryabhaṭa $=02^{\circ} 05^{\prime}$ and modern $\delta \lambda$ per day $=58^{\prime} 38^{\prime \prime}$.
- True solar ingress = 21 March 517 CE 06:00 ZT or 1321506 days -2.13 days $=1321503.9$ days.

It becomes apparent that the elements of Āryabhatīya yields as precisely the true equinox of 517 CE with the greater value of equation of centre. This is possible only if the treatise is based on the observation of the equinox under reference.
True longitude of sun could be determined from the midday shadow of gnomon ${ }^{13}$ as instructed by Bhāskara-I which of course is representative of the Āryabhaṭa tradition.

## दिनमध्यछायार्कादुच्चविशुद्धाद् भुजाफलं यत् स्यात्। तत् क्षयधनविपरीतादविशेषविधे रवेर्मध्यम् । |

"Subtract the apogee from the true longitude of sun derived using the midday shadow of the gnomon and calculate the equation of centre. Applying the same reversely the mean longitude is obtained from the true longitude. Mean longitude is then used to derive equation of centre and the same is applied to the true longitude and the process is repeated till the two successive results are the same".
 spring equinox of 19 March 517 CE and the meridian transit of the eclipsed moon on 23 March 517 CE. It is apparent from the deductions given above that Āryabhaṭa could check the longitude of sun from a measurement of midday shadow of sun and also by making use of the eclipsed moon transiting the meridian as explained earlier.

## III. Conclusions

1. Total lunar eclipse of 23 March 517 CE is shown to be a likely observation that had been of critical significance to Āryabhaṭa in fixing the elements of his immortal work Āryabhaṭīya. Significance of the meridian transit of the totally eclipsed moon has been brought out in the discussion.
2. Eclipse elements of Āryabhaṭīya is shown to be in precise agreement to the tune of minutes of arc with the results from modern astronomical algorithms in the case of mean longitudes of sun, moon, moon's apogee and node while the true longitude of sun is found to deviate by 9 minutes and moon by 30 minutes. Deviation has been explained as due to the peculiarity of the equation of centre employed in Indian astronomy.
3. Examination of the equinox close to the lunar eclipse suggests that the observation of the equinox of 517 CE on 19 March may have played a critical role in the fixing of solar years as 365.25868 days in Āryabhaṭiya.
4. Precise matching of the results of Āryabhatīya with modern results deduced for Camravattam (10N51, 75E45) reinforce the conclusions of earlier studies that Āryabhaṭa belonged to Kerala. Lack of reference to equation of time (Udayāntara and Bhujāntara) and

Deśāntara in Āryabhaṭīya renders additional support to the location of Āryabhaṭa in the low latitude of Kerala and on the meridian of Ujjayinī (75E45).

## IV. References

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\({ }^{12}\) Ibid., p. 112 presents a discussion on the historical development of equation of time in Indian
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\({ }^{13}\) Bhāskara-I outlines the method in Mahābhāskarīyam VIII.5. Ref. (9) above p. 49 and 217
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[^0]:    ${ }^{f}$ Danjon's five point scale for evaluating the visual appearance and brightness of the Moon during total lunar eclipses. 'L' values for various luminosities are defined as follows:
    $\mathrm{L}=0$ : Very dark eclipse. Moon almost invisible, especially at mid-totality.
    $\mathrm{L}=1$ : Dark Eclipse, gray or brownish in coloration. Details distinguishable only with difficulty.
    $\mathrm{L}=2$ : Deep red/rust-colored eclipse. Darker central shadow, outer edge of umbra is relatively bright.
    $\mathrm{L}=3$ Brick-red eclipse. Umbral shadow usually has a bright or yellow rim.
    $\mathrm{L}=4$ Very bright copper-red or orange eclipse. Umbral shadow has a bluish, very bright rim.

[^1]:    ${ }^{1}$ JD (TT):Mean Full Moon (Āryabhaṭīya) 1909973.89270370, 09:25 TT for 1909974.051, 13:13 ZT
    JD (TT): Mean Full Moon (modern) 1909973.894, 09:27TT for JD $(\mathrm{ZT})=1909974.0523,13: 15 \mathrm{ZT}$
    JD (TT): True Full Moon (Āryabhaṭ̂ya) 1909974.3283, 19:53 TT for JD $(\mathrm{ZT})=1909974.4866,23: 41 \mathrm{ZT}$ JD(TT): True Full Moon (modern): 1909974.3339, 20:01TT for JD (ZT) $=1909974.49218,23: 49$ ZT

